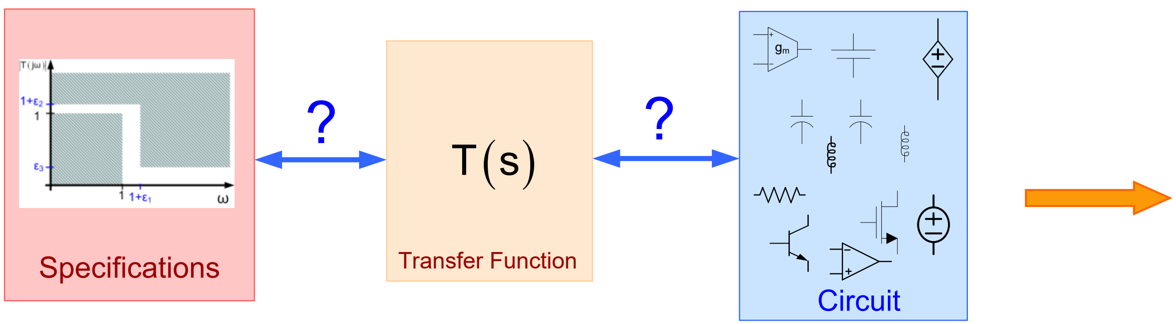
EE 508
Lecture 4

Filter Concepts/Terminology

Basic Properties of Electrical Circuits

Review from Last Time

Filter Design Process



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Establish Specifications
- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

↓

Approximation
- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

↓

Synthesis
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

↓

Filter

Review from Last Time

Biquadratic Factorization

If n is even and n≥m,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}=K\bullet\prod_{i=1}^{n/2} T_{BQi}(s)$$

If n is odd and n≥m,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}=K\bullet\left(\frac{a_{10}s+a_{00}}{s+b_{00}}\right)\bullet\prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

where $$T_{BQi}(s)=\frac{a_{2i}s^2+a_{1i}s+a_{0i}}{s^2+b_{1i}s+b_{0i}}$$

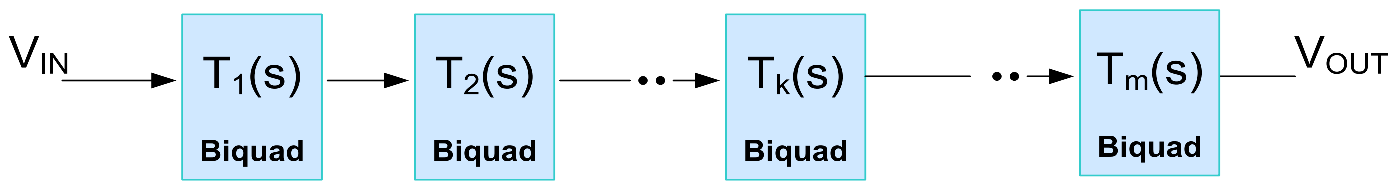
and where K is a real constant and all coefficients are real (some may be 0)

- Factorization is not unique
- H(z) factorizations not restricted to have m≤n
- Each biquatratic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator

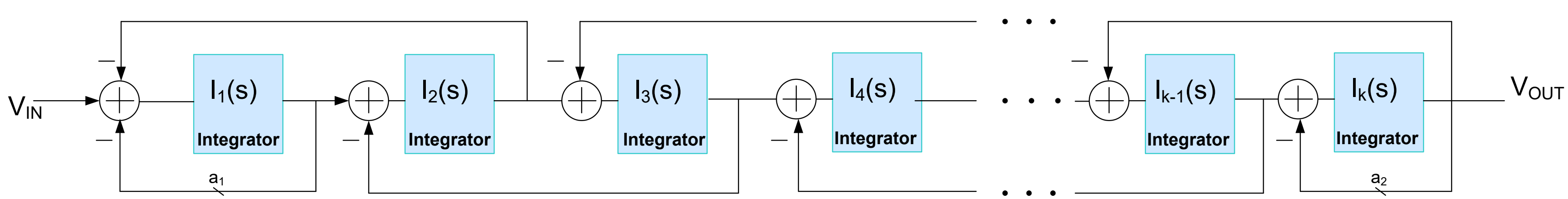
Review from Last Time

Common Filter Architectures

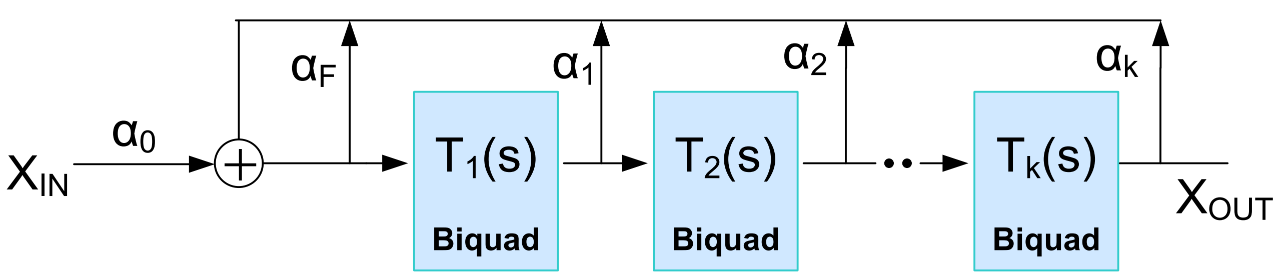
Cascaded Biquads



Leapfrog



Multiple-loop Feedback

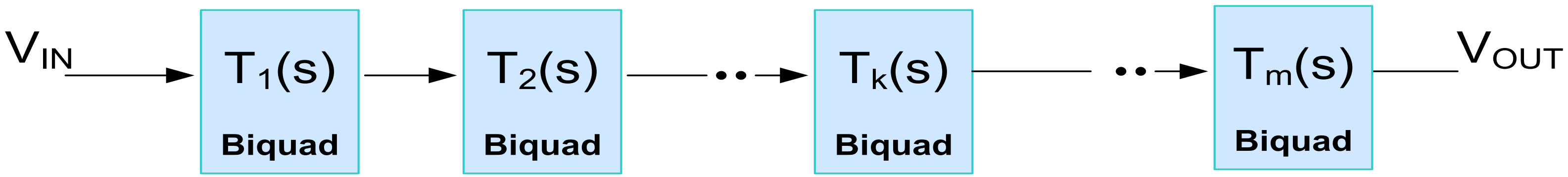


- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common
- The Cascaded Biquad structure follows directly from the Biquadratic Factorization

Review from Last Time

Common Filter Architectures

Cascaded Biquads



$$T(s) = T_1 T_2 \bullet \bullet T_m$$

- Sequence in Cascade often affect performance
- Different biquadratic factorizations will provide different performance
- Although some attention was given to the different alternatives for biquadratic factorization, a solid general formulation of the cascade sequencing problem or the biquadratic factorization problem never evolved

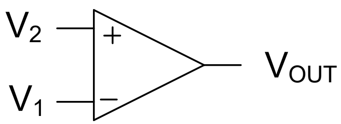
Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- ➡ Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

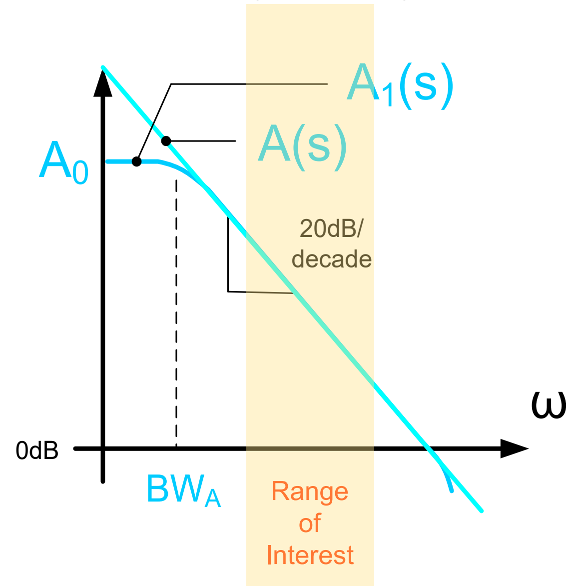
Gain, Bandwidth and GB

Frequency Dependent Model of Op Amps

Most op amps are designed so that they behave as a first-order circuit at frequencies up to the unity gain frequency or beyond



$$A_1(s)=\frac{V_{OUT}}{V_2-V_1}$$



$$A_1(s)=\frac{A_0}{\frac{s}{BW_A}+1} \Leftrightarrow A_1(s)=\frac{GB}{s+BW_A}$$

$$GB=A_0 \bullet BW_A$$

$$BW_A=\frac{GB}{A_0}<1$$

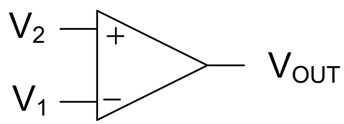
Can usually model with a more-simplified gain expression in frequency region of interest

$$A(s)=\frac{GB}{s}$$

Adequate model for most applications

Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

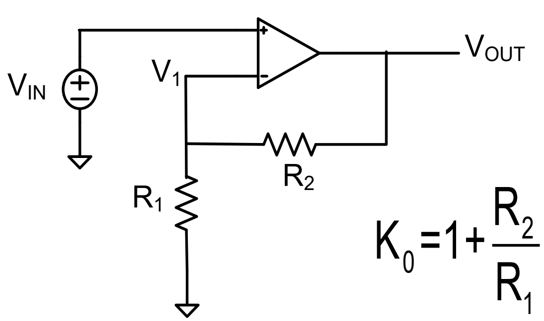
$$\left.\begin{aligned} V_1 &= \frac{V_{OUT}}{K_0} \\ V_{OUT} &= A_1(s)(V_{IN} - V_1) \\ A_1(s) &= \frac{GB}{s + BW_A} \end{aligned}\right\}$$

$$A_{FB}(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_0}{s\frac{K_0}{GB} + \left(1 + K_0\frac{BW_A}{GB}\right)}$$

Same as using “adequate” model

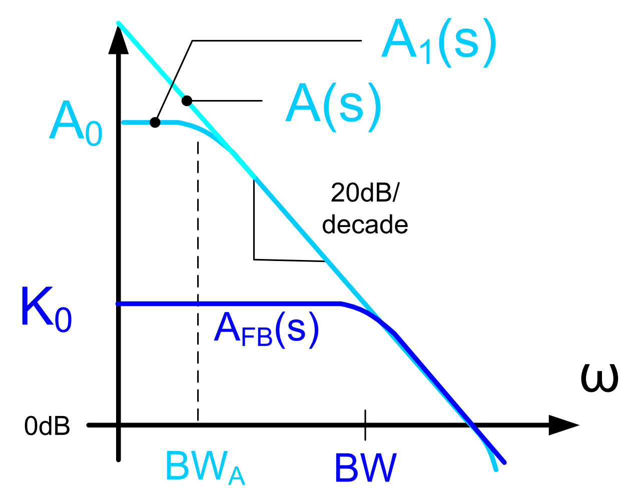
$$A_{FB}(s) \cong \frac{K_0}{1 + s\frac{K_0}{GB}}$$

$$BW = \frac{GB}{K_0}$$



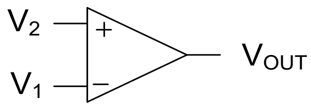
$$K_0 = 1 + \frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

Adequate model for most applications

$$A_{FB}(s) \cong \frac{K_0}{1+s\frac{K_0}{GB}}$$

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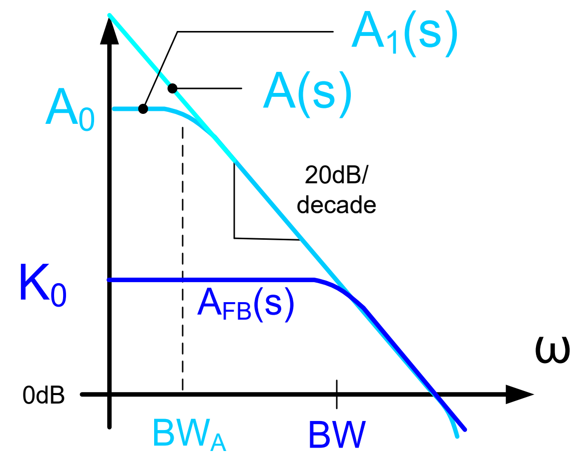
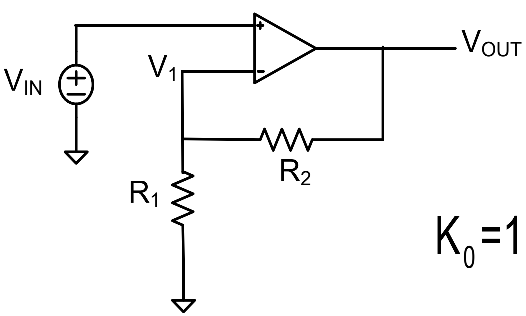
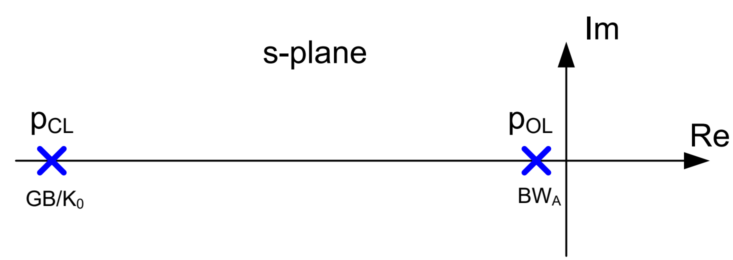




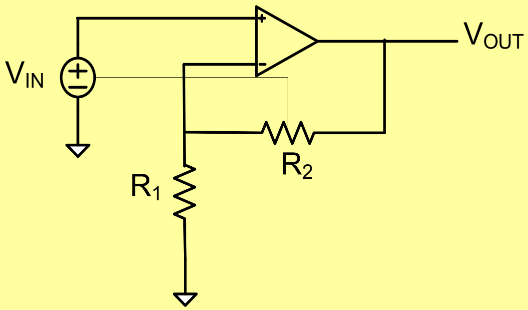
$$K_0=1+\frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

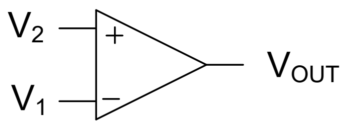


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s\frac{K_0}{GB}}$$

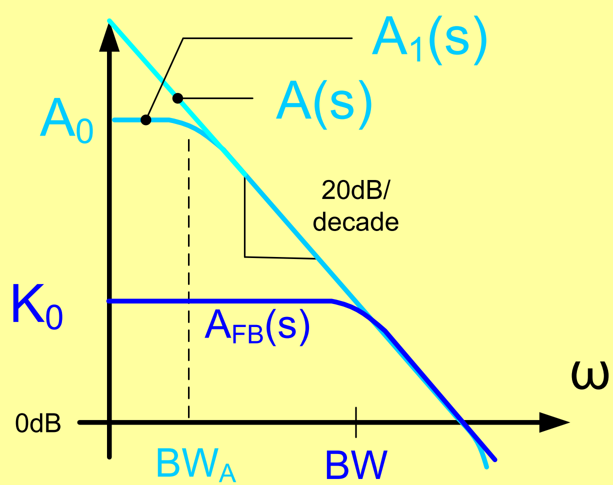


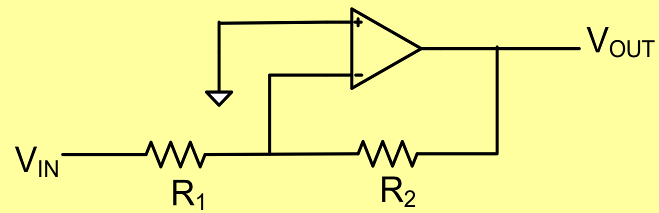
$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications





Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

$$A_{FB}(s) = -\frac{K_0}{1 + s\frac{(1 + K_0)}{GB}}$$

Filter Concepts and Terminology

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- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
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Stability and Instability

True or False?

An unstable circuit will oscillate

False – unstable circuits will either latch up or oscillate. Latch-up is often the consequence of saturating nonlinearities of circuits that have positive real axis poles

Achieving stability is a major goal of the filter designer

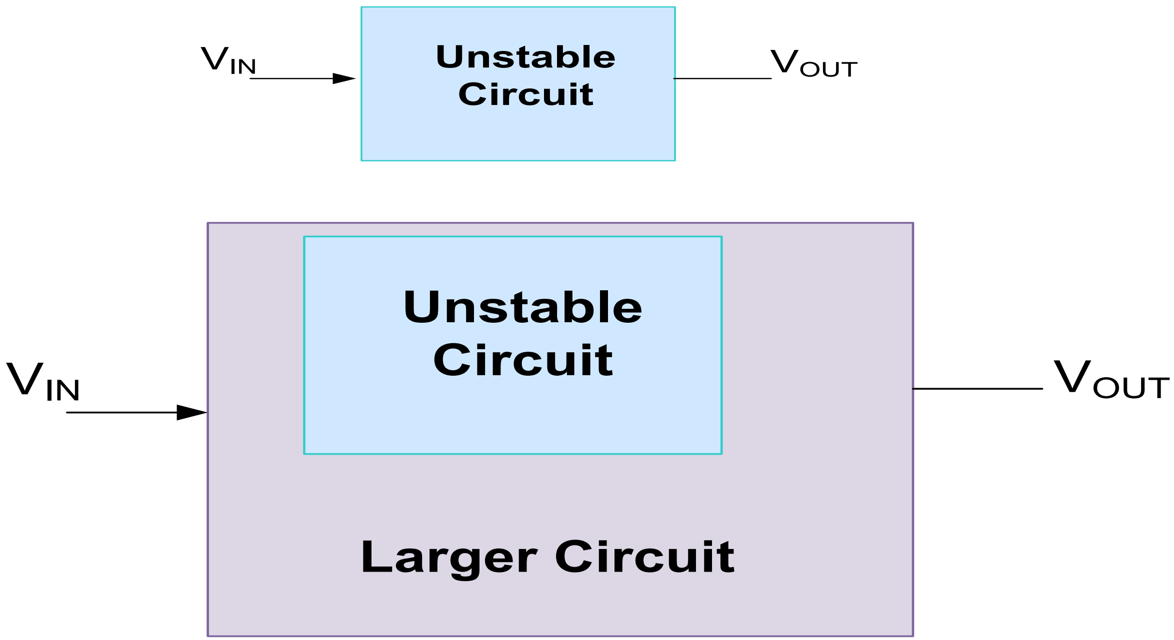
False – a filter is usually of little practical use if there are concerns about stability

Unstable circuits are of little use in designing filters

False – will discuss details later

Theorem ?:

If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

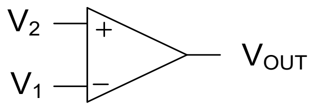


Proof ?:

Consider First Some Related Concepts

Gain, Bandwidth and GB

Consider "positive feedback" closed-loop amplifier

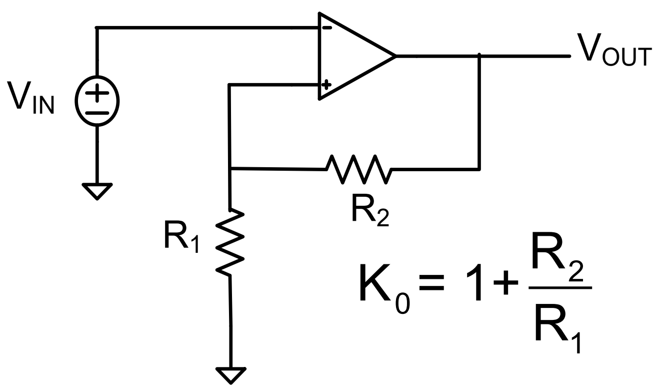


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

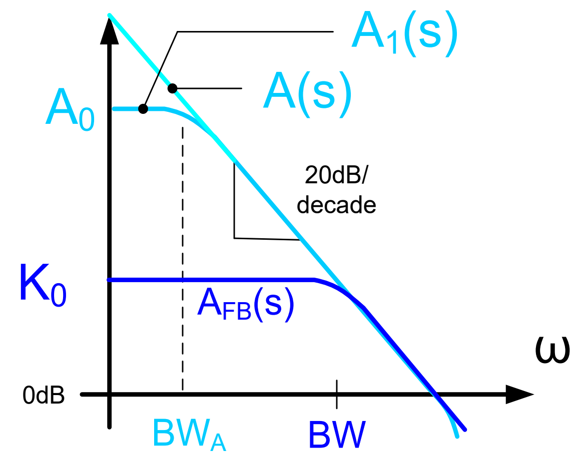
Adequate model for most applications

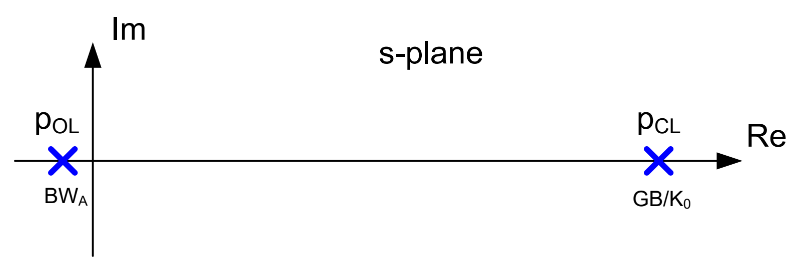


$$K_0 = 1 + \frac{R_2}{R_1}$$

$$A_{FB}(s) \cong \frac{K_0}{1 - s\dfrac{K_0}{GB}}$$

$$BW = \frac{GB}{K_0}$$

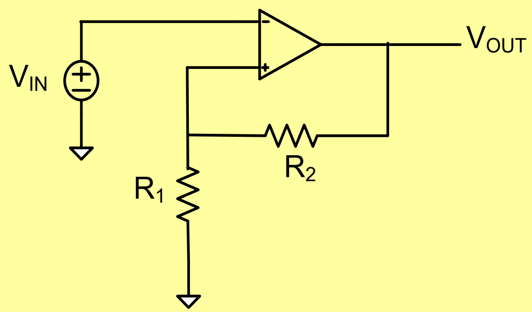




Feedback Amplifier is Unstable !

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with "Positive Feedback"

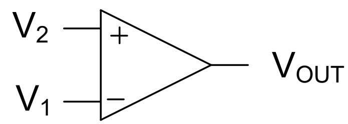


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 - s\frac{K_0}{GB}}$$



$$A_1(s) = \frac{GB}{s + BW_A}$$

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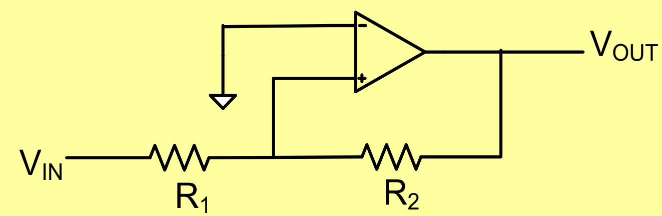
Adequate model for most applications



Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

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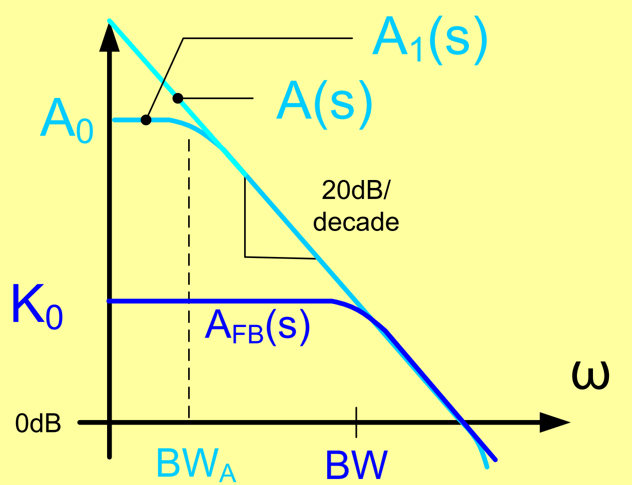
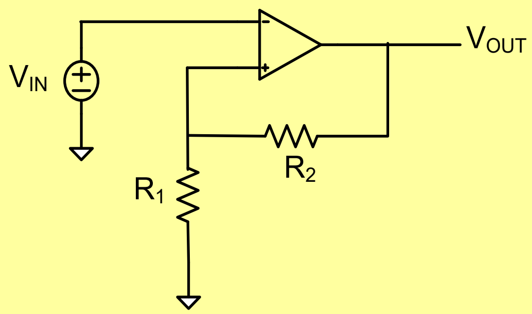


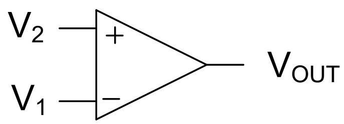
Both FB Amplifiers are Unstable

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with "Positive Feedback"



Basic Noninverting Amplifier

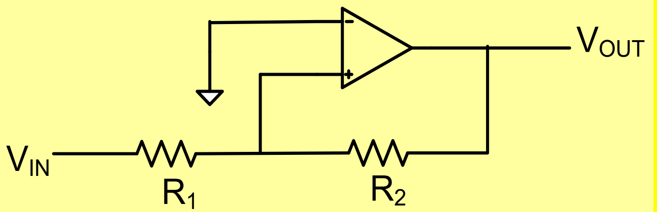


$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

Adequate model for most applications



Basic Inverting Amplifier

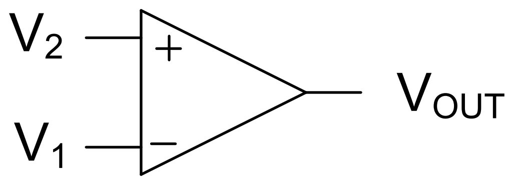
Both FB Amplifiers are Unstable

Is "Positive Feedback" bad?

- Engineers often make the assumption that positive feedback is bad and must be avoided
- Positive feedback in these stand-alone amplifiers resulted in unstable circuits
- Positive feedback is often very beneficial and should not be unilaterally avoided

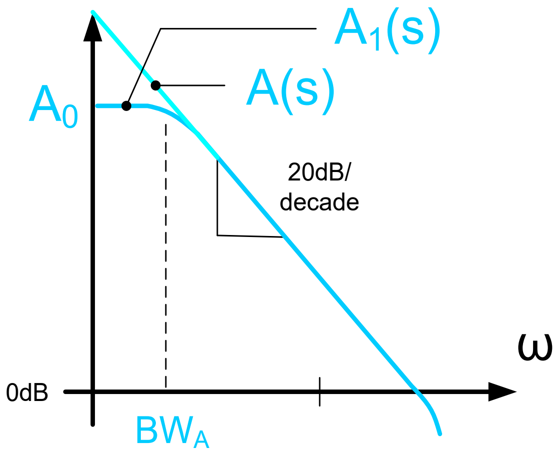
Gain, Bandwidth and GB

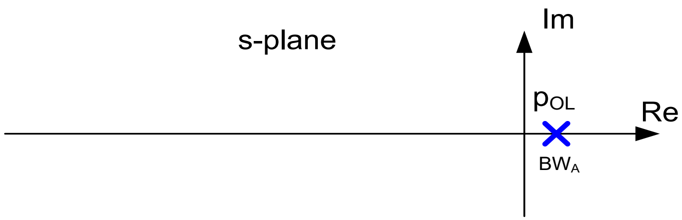
Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s) = \frac{GB}{s - BW_A}$$

$$|GB| = A_0 \bullet BW_A$$

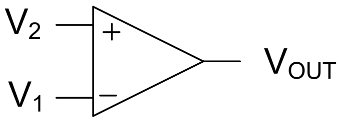




Op Amp is Unstable, dc gain is negative

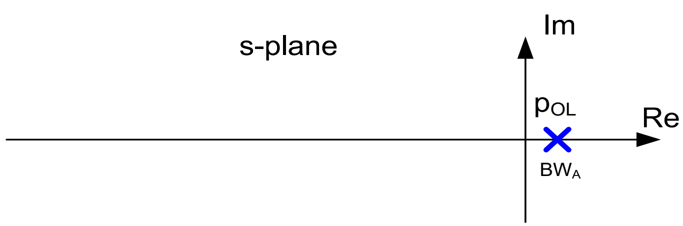
Gain, Bandwidth and GB

Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s)=\frac{GB}{s - BW_A}$$

$$|GB| = A_0 \bullet BW_A$$

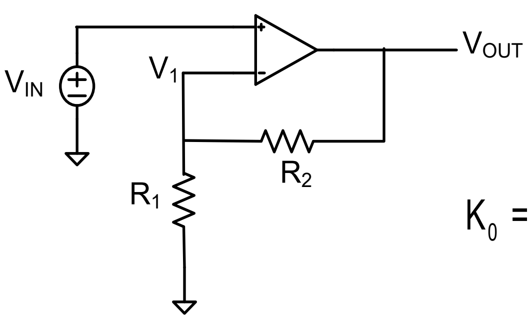


$$\left.\begin{aligned} V_1 &= \frac{V_{OUT}}{K_0} \\ V_{OUT} &= A(s)(V_{IN}-V_1) \\ A(s) &= \frac{GB}{s-BW_A} \end{aligned}\right\}$$

$$A_{FB}(s)=\frac{V_{OUT}}{V_{IN}}=\frac{K_0}{s\frac{K_0}{GB}+\left(1-K_0\frac{BW_A}{GB}\right)}$$

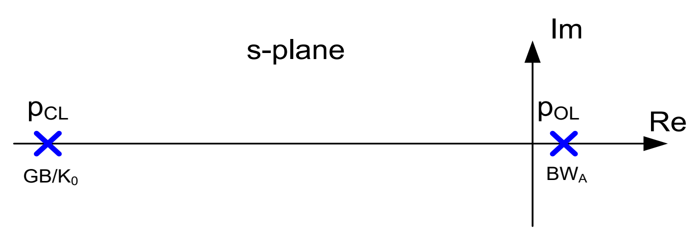
$$A_{FB}(s)\cong\frac{K_0}{1+s\frac{K_0}{GB}}$$

$$BW=\frac{GB}{K_0}$$



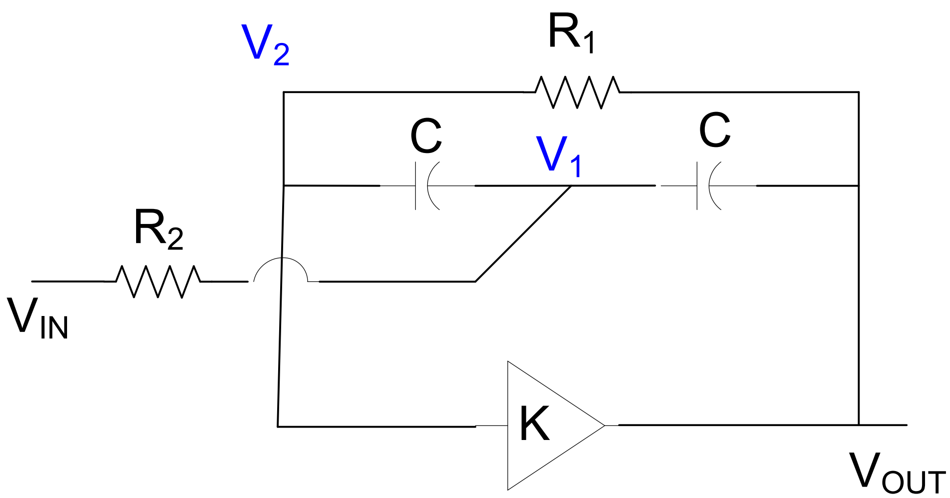
Basic Noninverting Amplifier

$$K_0 = 1+\frac{R_2}{R_1}$$



- Feedback Amplifier is stable and performs very well!
- Serves as counter-example for “Theorem”!

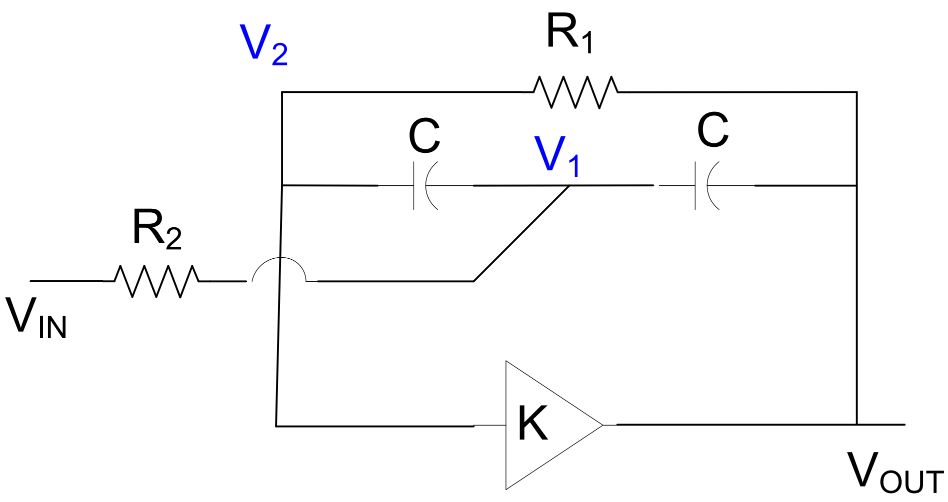
Consider another Filter Example:



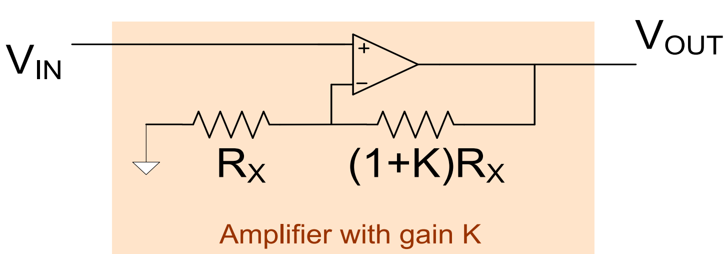
$$\left.\begin{aligned} V_1\left(sC+sC+G_2\right) &= V_{IN}G_2+V_2sC+V_{OUT}sC \\ V_2\left(sC+G_1\right) &= V_1sC+V_{OUT}G_1 \\ V_{OUT} &= KV_2 \end{aligned}\right\}$$

$$T(s) = \frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$

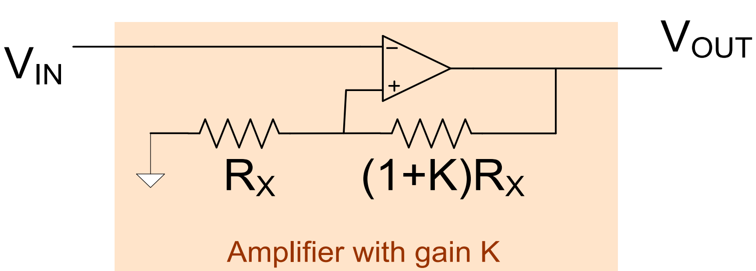
Consider Filter Example:



$$T(s) = \frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$



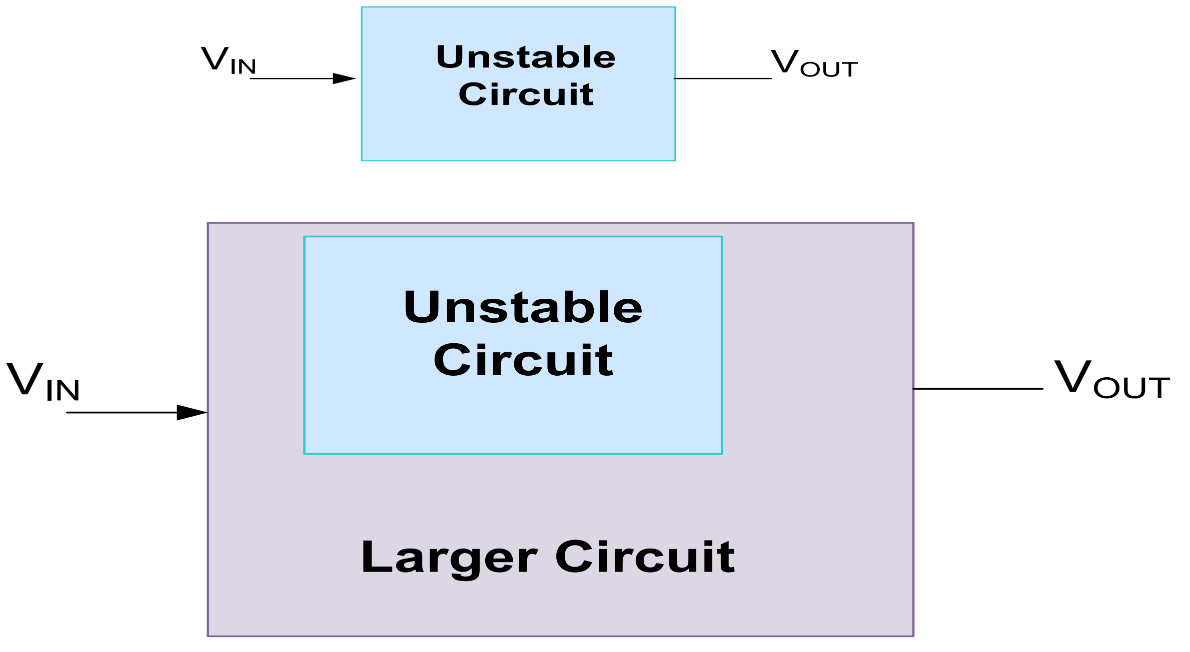
- Stable Amplifier
- But if used in above, filter will be unstable



- Unstable Amplifier
- But if used in above, filter will be stable
- Serves as another counter example for "theorem"

Theorem:

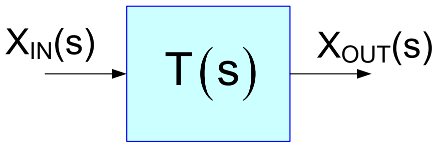
If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

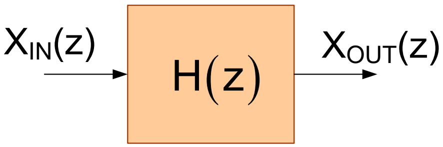


Proof:

This theorem is not valid though many circuit and filter designers believe it to be true !

Filter Concepts and Terminology





Stability Issues:

Is stability or instability good or bad?

Often there is an impression that instability is bad - but why?

Some observations:

- An unstable filter does not behave as a filter
- Unstable filter circuits are often used as waveform generators
- If an unstable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- If a stable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- Digital latches, RAMs, etc. are unstable amplifiers
- Some of the best filter circuits include an embedded unstable filter

Stability or Instability is neither good or bad, but it is important for the designer to be aware of the opportunities and limitations associated with this issue

Filter Concepts and Terminology

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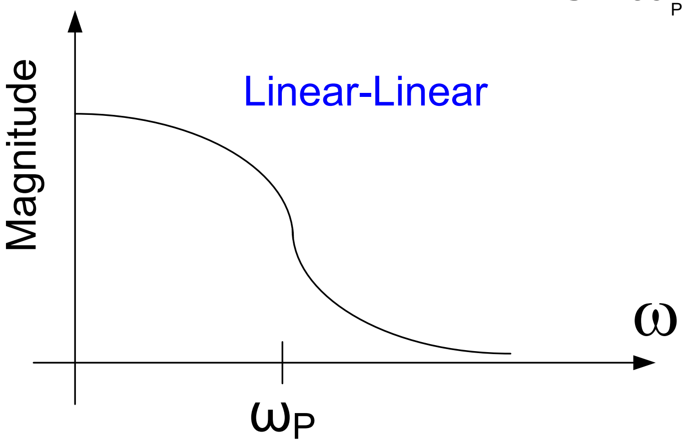
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

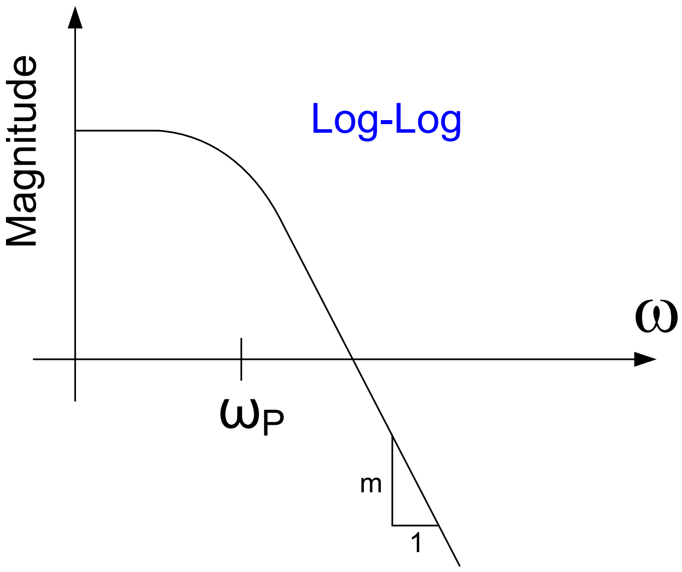
Single-pole roll-off characterization

Consider: $T(s) = \dfrac{\omega_P}{s+\omega_P}$

$$T(j\omega) = \frac{\omega_P}{j\omega+\omega_P}$$



$$\left|T(j\omega)\right| = \frac{\omega_P}{\sqrt{\omega^2+\omega_P^2}}$$



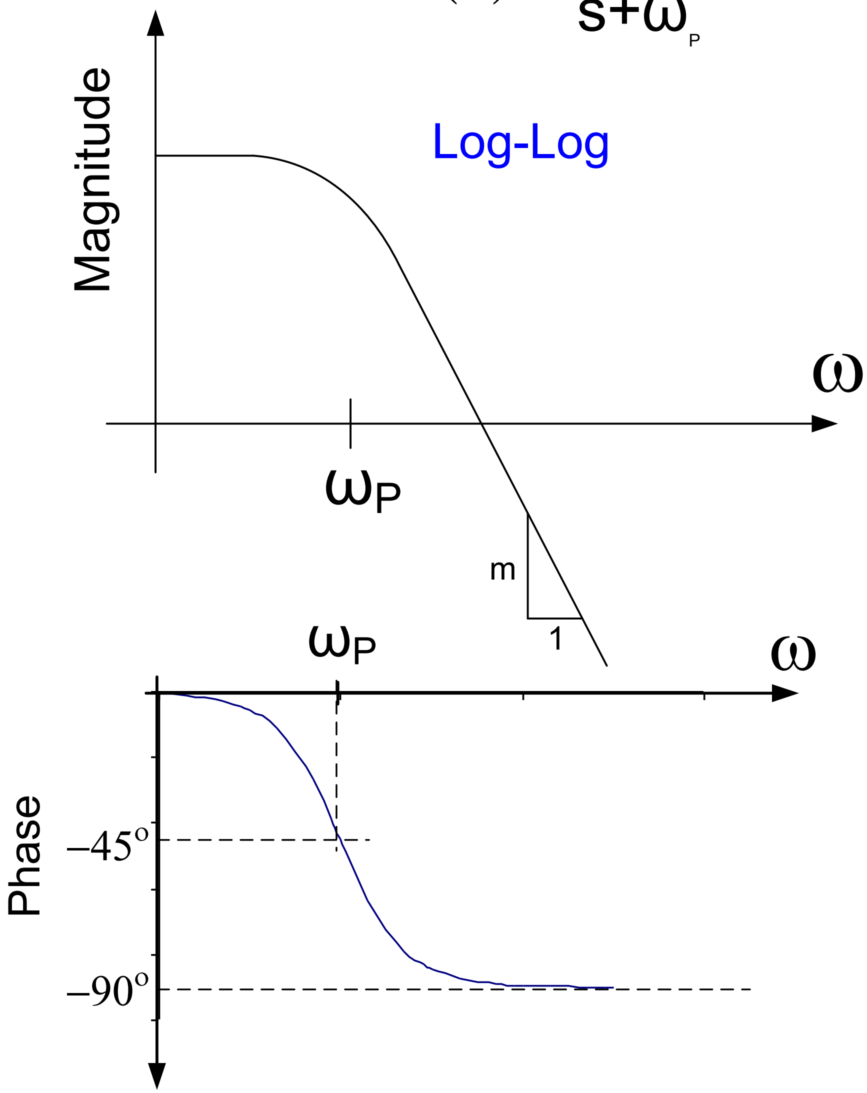
m = - 20dB/decade

m = -6dB/octave

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_P}\right)$$

Single-pole roll-off characterization

Consider: $T(s) = \dfrac{\omega_P}{s+\omega_P}$



$$T(j\omega) = \frac{\omega_P}{j\omega+\omega_P}$$

$$\left|T(j\omega)\right| = \frac{\omega_P}{\sqrt{\omega^2+\omega_P^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_P}\right)$$

Roll-off characterization

At frequencies well-past a pole or zero, each LHP pole (real or complex) causes a roll-off in magnitude on a log-log axis of -20dB/decade and each LHP zero causes a roll-off of +20dB/decade

At frequencies of magnitude comparable to that of a pole or zero, it is not easy to predict the roll-off in the magnitude characteristics by some simple expression

Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- ➡ Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

Distortion in Filters

- Magnitude Distortion
  - frequency dependent change in gain of a circuit (usually bad if building amplifier but critical if building a filter)
- Phase Distortion
  - a circuit has phase distortion if the phase of the transfer function is not linear with frequency
- Nonlinear Distortion
  - Presence of frequency components in the outut that are not present in the input (generally considered bad in filters but necessary in many other circuits)

Stay Safe and Stay Healthy !

End of Lecture 4